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The Axiomatic Foundations of the Theory of Special Relativity

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Abstract

In this paper it is shown that (1) linear transformations more general than the Lorentz transformation—containing the Palacios and the Lorentz transformation as special cases—(2) and the 'principle' of the constancy of the velocity of light (taken originally by Einstein together with the supposition of the linearity of transformation as fundamental hypotheses of the theory of special relativity)—can be deduced from Maxwell's equations for the electromagnetic field *in vacuo* (A_1), the principle of relativity (A_2) and the two following axioms (which do not contain explicitly the hypothesis of the isotropy of space!): (A_3) to every event in the Galilean reference system S there corresponds one and only one event in the system S' so that these two systems are connected by reversible single-valued functions, continuously differentiable as their inverse transformations, (A_4) the constant relative velocities $v_{ss'}$ and $v_{s's}$ between S and S' are each other equal in magnitude and opposite in sign $v_{ss'} = -v_{s's}$. To obtain uniquely the Lorentz transformation the following axiom has to be added: (A_5) the distance D of any two points at rest in S, situated in a plane orthogonal to the relative velocity between S and S' is measured S' as independent of the sense of the velocity, i.e. if one changes $v_{ss'}$ into $-v_{ss'}$ the distance D does not vary for an observer in S'. Results of our theory are the ideas that (a) the fact that the Lorentz transformation is not the unique transformation leaving Maxwell's equations for the electromagnetic field in all Galilean systems of reference invariant but that there exists a more general transformation (containing these two transformations as special cases) leaving Maxwell's equations invariant; (b) that the Michelson–Morley as well as the Fizeau experiment does not represent an experimental proof in favour of the theory of special relativity. At the end of the paper the mutual relations between the principle of relativity (the axiom A_1 together with the axiom A_2), the axiom A_5 and the possibility of the discernibility as well as the indiscernibility of 'right' and 'left' at the macrocosmic level is discussed.

1. *Introduction*

The foundations of mathematics is today a fertile and well-established branch of mathematics. The foundations of physics are in their first beginnings: the investigations concerning the logical structure, such as, for example, the question of the mutual dependence or independence of some basic statements of the majority of the physical theories, are in their embryonic or at best at a protoscientific level. Foundations research consists in the elimination of logical inconsistencies by means of the axiomatic method. At a certain point in the development every theory becomes mature for axiomatization. By axiomatization of a theory the logically unperspicuous set of propositions becomes well organized, the structure of a theory becomes clear and perspicuous (Bunge, 1967). The present paper is a contribution to the foundations of the theory of special relativity. It issues from the author's critical study of Einstein's 1905 paper 'Zur Elektrodynamik bewegter Körper' and the fundamental axiomatic problem if the theory of special relativity could not be founded without the supposition of the linearity of transformation connecting any two Galilean systems of reference and the 'principle' of the constancy of the velocity of light starting from Maxwell's equations for the electromagnetic field *in vacuo,* the principle of relativity and some other basic axioms of physics. The results of the author's former investigations obtained in 1962 and presented in his Doctor-thesis at the Technical University of Munich (Stiegler, 1963) are, in the present paper, generalized and enlarged.

2. *The Set of Basic Axioms*

Our considerations are based on the following set of four axioms:

A_1. For the electromagnetic phenomena *in vacuo* Maxwell's equations

$$\operatorname{rot}\mathfrak{E} = -\frac{1}{c}\frac{\partial\mathfrak{H}}{\partial t} \qquad \operatorname{rot}\mathfrak{H} = \frac{1}{c}\frac{\partial\mathfrak{E}}{\partial t} \qquad \operatorname{div}\mathfrak{E} = 0 \qquad \operatorname{div}\mathfrak{H} = 0$$

are valid, c being the velocity of light *in vacuo.*

A_2. In all Galilean systems of reference the physical laws have the same analytical form (principle of relativity).

A_3. To every event in the Galilean system of reference S there corresponds one and only one event in the Galilean system of reference S' and conversely, so that the coordinates and the times of these two systems are connected by reversible single-valued functions

$$x_i' = f_i(v, x_0, x_1, x_2, x_3) \quad i = 0, 1, 2, 3$$

These functions and their corresponding inverse transformations are continuously differentiable, where

$$x_0 = ct \qquad x_1 = x \qquad x_2 = y \qquad x_3 = z$$

and

$$x_0' = c' t' \qquad x_1' = x' \qquad x_2' = y' \qquad x_3' = z'$$

v being the relative velocity of the systems of reference S and S', c' the velocity of light in the system S', for which we do not know *a priori* if it is equal to the constant c of the velocity of light in the system S.

A_4. The constant relative velocities $v_{ss'}$ and $v_{s's}$ of the Galilean systems of reference S and S' are to each other oppositely equal

$$v_{ss'} = -v_{s's}$$

3. *Consequences from the Given Set of Basic Axioms*

Let $A_1, A_2, \ldots, A_n$ be a set of propositions of a theory τ. Then one of the fundamental problems is the question on the independency of the set of propositions $A_1, A_2, \ldots, A_n$. The proof of independency of a set of propositions $A_1, A_2, \ldots, A_n$ of a theory τ consists in that it will be shown that the proposition A_k for any $k \in \{1,2,\ldots,n\}$ is not a theorem within τ, where the propositions A_l, $l \neq k$, are taken as axioms of τ. If one succeeds in showing that there exists a unique proposition A_k which is a theorem in τ, the other propositions A_l, $l \neq k$, being axioms of τ, then such a system of propositions $A_1, A_2, \ldots, A_k, \ldots, A_n$ will not be independent (Bourbaki, 1960).

In the following it shall be proved concretely that the linearity of transformation, the 'principle' of the constancy of the velocity of light and consequently the more general transformation (3.3.2) are theorems in our theory which is based on the axioms A_1, A_2, A_3 and A_4.

A surprising consequence of our theory is the result that the axioms A_1–A_4 conduce to a more general transformation (3.3.2) which leaves Maxwell's equations for the electromagnetic field invariant, containing the Lorentz and the Palacios transformation as special cases. Thus the theory based on the set of axioms A_1, A_2, A_3 and A_4, given in Section 2, contains the theory of special relativity and the Palacios theory as special cases.

3.1. *The Linearity of Transformation*

For the following considerations the equation for the propagation of the electromagnetic wave-front

$$\frac{1}{c^2}\left(\frac{\partial \Omega}{\partial t}\right)^2 - (\operatorname{grad}\Omega)^2 = 0 \tag{3.1.1}$$

has a fundamental importance, $\Omega(x,y,z,t) = 0$ being the equation of the wave surface. This equation follows directly from Maxwell's equations for the electromagnetic field *in vacuo* (A_1). Equation (3.1.1) can be reduced to the form of the Hamilton–Jacobi differential equation of classical mechanics (Fock, 1960a).

$$\frac{\partial \Omega}{\partial t} + c\sqrt{(\Omega_x^2 + \Omega_y^2 + \Omega_z^2)} = 0 \tag{3.1.2}$$

The quantity corresponds to the action function S, the derivatives Ω_x, Ω_y, Ω_z to the momenta p_x, p_y, p_z, the function

$$H = c\sqrt{(\Omega_x^2 + \Omega_y^2 + \Omega_z^2)} \tag{3.1.3}$$

to the Hamilton function and the light rays to the trajectories of material particles. The system of characteristic differential equations corresponding to the equation

$$\frac{\partial \Omega}{\partial t} + H(\Omega_x, \Omega_y, \Omega_z) = 0 \tag{3.1.4}$$

is given by

$$\frac{dt}{1}=\frac{dx}{\partial H/\partial \Omega_x}=\frac{dy}{\partial H/\partial \Omega_y}=\frac{dz}{\partial H/\partial \Omega_z}$$
$$=\frac{d\Omega}{\Omega_t+\Omega_x(\partial H/\partial \Omega_x)+\Omega_y(\partial H/\partial \Omega_y)+\Omega_z(\partial H/\partial \Omega_z)}=\frac{d\Omega_t}{-\partial H/\partial t}$$
$$=\frac{d\Omega_x}{-\partial H/\partial x}=\frac{d\Omega_y}{-\partial H/\partial y}=\frac{d\Omega_z}{-\partial H/\partial z} \tag{3.1.5}$$

The differential equations for the rays are then given by

$$\frac{dx}{dt}=\frac{\partial H}{\partial \Omega_x}=c\frac{\Omega_x}{\sqrt{(\Omega_x{}^2+\Omega_y{}^2+\Omega_z{}^2)}} \qquad \frac{dy}{dt}=\frac{\partial H}{\partial \Omega_y}=c\frac{\Omega_y}{\sqrt{(\Omega_x{}^2+\Omega_y{}^2+\Omega_z{}^2)}}$$
$$\frac{dz}{dt}=\frac{\partial H}{\partial \Omega_z}=c.\frac{\Omega_z}{\sqrt{(\Omega_x{}^2+\Omega_y{}^2+\Omega_z{}^2)}} \tag{3.1.6a}$$

$$\frac{d\Omega_x}{dt}=-\frac{\partial H}{\partial x}=0 \qquad \frac{d\Omega_y}{dt}=-\frac{\partial H}{\partial y}=0 \qquad \frac{d\Omega_z}{dt}=-\frac{\partial H}{\partial z} \tag{3.1.6b}$$

These equations are analogous to Hamilton's equations of classical mechanics. From (3.1.6b) it is evident that Ω_x, Ω_y and Ω_z are constant along a ray. The equations of rays are then

$$x-x_0=c.\frac{\Omega_x}{\sqrt{(\Omega_x{}^2+\Omega_y{}^2+\Omega_z{}^2)}}(t-t_0)$$
$$y-y_0=c.\frac{\Omega_y}{\sqrt{(\Omega_x{}^2+\Omega_y{}^2+\Omega_z{}^2)}}(t-t_0)$$
$$z-z_0=c.\frac{\Omega_z}{\sqrt{(\Omega_x{}^2+\Omega_y{}^2+\Omega_z{}^2)}}(t-t_0) \tag{3.1.7}$$

and that means that the light is propagated rectilinearly with constant velocity.

If $\Omega^0(x_0, y_0, z_0)=0$ represents the wave-surface corresponding to a definite instant t_0 and x_0, y_0, z_0 are its coordinates, for the direction cosines of the normal to this surface we have

$$\alpha(x_0, y_0, z_0)=\frac{\Omega_x{}^0}{\sqrt{(\Omega_x^{0\,2}+\Omega_y^{0\,2}+\Omega_z^{0\,2})}}$$
$$\beta(x_0, y_0, z_0)=\frac{\Omega_y{}^0}{\sqrt{(\Omega_x^{0\,2}+\Omega_y^{0\,2}+\Omega_z^{0\,2})}}$$
$$\gamma(x_0, y_0, z_0)=\frac{\Omega_z{}^0}{\sqrt{(\Omega_x^{0\,2}+\Omega_y^{0\,2}+\Omega_z^{0\,2})}} \tag{3.1.8}$$

and consequently for equations of a ray going through the point (x_0, y_0, z_0) of the initial wave-surface

$$x - x_0 = \alpha c(t - t_0)$$
$$y - y_0 = \beta c(t - t_0)$$
$$z - z_0 = \gamma c(t - t_0) \tag{3.1.9}$$

From (3.1.7) and (3.1.9) there follows

$$c^2(t - t_0)^2 - [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2] = 0 \tag{3.1.10}$$

According to the axioms A_1 and A_2 in the Galilean system of reference S' we have

$$\text{rot}' \mathfrak{E}' = -\frac{1}{c'}\frac{\partial \mathfrak{H}'}{\partial t'} \qquad \text{rot}' \mathfrak{H}' = \frac{1}{c'}\frac{\partial \mathfrak{E}'}{\partial t'} \qquad \text{div}' \mathfrak{E}' = 0 \qquad \text{div}' \mathfrak{H}' = 0$$

and consequently

$$c'^2(t' - t_0')^2 - [(x' - x_0')^2 + (y' - y_0')^2 + (z' - z_0')^2] = 0 \tag{3.1.10'}$$

c' being the velocity of light in the system S', for which we do not know *a priori* if it equals the constant c in the system S.

In differential form we have

$$c^2\,dt^2 - (dx^2 + dy^2 + dz^2) = 0 \tag{3.1.11}$$

$$c'^2\,dt'^2 - (dx'^2 + dy'^2 + dz'^2) = 0 \tag{3.1.11'}$$

Relations (3.1.10) and (3.1.10′) between the coordinates of the initial and the final point of any ray represents a sphere in S and S' with the centre (x_0, y_0, z_0) and (x_0', y_0', z_0') and the radius $R = c.(t - t_0)$ and $R' = c'.(t' - t_0')$ respectively, expressing the fact that in every inertial system of reference taken as a whole the velocity of light is equal in all directions.

Besides this, from (3.1.7) and (3.1.9) it follows that the light is propagated rectilinearly with constant velocity. According to the axiom A_2 this is valid in all Galilean systems of reference.

With the aid of the quantities

$$e_0 = 1 \qquad e_1 = e_2 = e_3 = -1 \tag{3.1.12a}$$

and the symmetrical notations

$$x_0 = ct \qquad x_1 = x \qquad x_2 = y \qquad x_3 = z \tag{3.1.12b}$$

one can write the equations (3.1.11) and (3.1.11′) in the form

$$e_i\,dx_i\,dx_i = 0 \tag{3.1.13}$$

and $\qquad (i = 0, 1, 2, 3)$

$$e_i\,dx_i'\,dx_i' = 0 \tag{3.1.13'}$$

respectively, summation on index i.

The coordinates and the times of both the Galilean systems of reference S and S' are connected by the functions (axiom A_3)

$$x_i' = f_i(v, x_0, x_1, x_2, x_3) \qquad (i = 0, 1, 2, 3) \tag{3.1.14}$$

From the supposition about the functions f_i made in the axiom A_3—namely, that the transformation (3.1.14) is single-valued and reversible and that (3.1.14) and the inverse transformation corresponding to it is continuously differentiable—it follows that the Jacobi determinant is different from zero

$$\left|\frac{\partial f_i}{\partial x_k}\right| \neq 0 \tag{3.1.14'}$$

In accordance to our set of basic axioms A_1, A_2, A_3 and A_4 and consequently to the relation (3.1.14′) we shall determine the functions (3.1.14).

From (3.1.14) results

$$dx_i' = \frac{\partial f_i}{\partial x_k} dx_k \qquad \begin{matrix}(i = 0, 1, 2, 3)\\(k = 0, 1, 2, 3)\end{matrix} \tag{3.1.15}$$

and from (3.1.13′) and (3.1.15)

$$e_i\, dx_i'\, dx_i' = e_i \frac{\partial f_i}{\partial x_k} \frac{\partial f_i}{\partial x_l} dx_k\, dx_l = 0 \tag{3.1.16}$$

On the other hand

$$e_i\, dx_i\, dx_i = e_k\, \delta_{kl}\, dx_k\, dx_l = 0 \tag{3.1.17}$$

δ_{kl} being the Kronecker symbol.

From (3.1.16) and (3.1.17) then follows

$$e_i \frac{\partial f_i}{\partial x_k} \frac{\partial f_i}{\partial x_l} = \lambda e_k\, \delta_{kl} \tag{3.1.18}$$

where λ is a function of the relative velocity v of the Galilean systems of reference S and S' and coordinates x_0, x_1, x_2 and x_3,

$$\lambda = \lambda(v, x_0, x_1, x_2, x_3)$$

which must be determined. This being the first condition for the functions f_i.

Now we denote by ξ_0, ξ_1, ξ_2, ξ_3 the arbitrary initial values of the coordinates x_0, x_1, x_2 and x_3 and introduce the parameter

$$s = \frac{c}{\beta_0}(t - t_0) \tag{3.1.19}$$

and the quantities

$$\beta_i = \frac{v_i}{c} \qquad (i = 1, 2, 3) \tag{3.1.20}$$

where $\beta_0 > 0$ and v_i are the components of the constant relative velocity v of the Galilean system of reference S and S'. A rectilinear uniform motion with respect to the system S can be represented by

$$x_i = \xi_i + \beta_i . s \qquad (i = 0, 1, 2, 3) \tag{3.1.21}$$

Since a rectilinear motion with respect to the system S will be also a rectilinear one relative to a Galilean system of reference S', we have

$$x_i' = \xi_i' + \beta_i' s' \tag{3.1.21'}$$

From (3.1.21') one gets

$$\frac{dx_i'}{dx_0'} = \frac{\beta_i'}{\beta_0'} = \frac{(df_i/ds)\,ds}{(df_0/ds)\,ds} = \frac{\sum_{k=0}^{3} \beta_k(\partial f_i/\partial x_k)}{\sum_{k=0}^{3} \beta_k(\partial f_0/\partial x_k)} = \text{const.} \tag{3.1.22}$$

and therefrom by logarithmic derivation with respect to the parameter s

$$\frac{\sum_{k,l=0}^{3} \beta_k\beta_l(\partial^2 f_0/\partial x_k\,\partial x_l)}{\sum_{k=0}^{3} \beta_k(\partial f_0/\partial x_k)} = \frac{\sum_{k,l=0}^{3} \beta_k\beta_l(\partial^2 f_0/\partial x_k\,\partial x_l)}{\sum_{k=0}^{3} \beta_k(\partial f_i/\partial x_k)} \tag{3.1.23}$$

These relations must be identically satisfied in β_i and x_i.

Taking into account, that according to (3.1.14'), the Jacobi functional determinant is different from zero

$$\left|\frac{\partial f_i}{\partial x_k}\right| \neq 0$$

and that, on the other hand, β_i, because of the rectilinear uniform motion ($v_i \neq 0$) of both the Galilean systems of reference S and S', according to (3.1.20), are also different from zero, there follows, in accordance with a fundamental theorem of algebra, that all determinants in (3.1.23) cannot be simultaneously equal to zero. Consequently, all the fractions must then have always finite values, even if a denominator equals zero. But this is only possible if in these fractions the numerators are divisible by the corresponding denominators, i.e. if the expressions in (3.1.23) are not at all fractions but integer rational functions of β_i, for which we write

$$\frac{\sum_{k,l=0}^{3} \beta_k\beta_l(\partial^2 f_i/\partial x_k\,\partial x_l)}{\sum_{k=0}^{3} \beta_k(\partial f_i/\partial x_k)} = 2\sum_{l=0}^{3} \beta_l \chi_l \tag{3.1.24}$$

χ_l being in general functions of x_0, x_1, x_2, x_3 and v.

$$\chi_l = \chi_l(v, x_0, x_1, x_2, x_3)$$

Since the last relation is an identity in β_i the following holds

$$\frac{\partial^2 f_i}{\partial x_k\,\partial x_l} = \chi_k \frac{\partial f_i}{\partial x_l} + \chi_l \frac{\partial f_i}{\partial x_k} \tag{3.1.25}$$

We now differentiate (3.1.18) with respect to x_m and put

$$\frac{\partial \lambda}{\partial x_m} = 2\Phi_m \tag{3.1.26}$$

Φ_m being generally a function of v, x_0, x_1, x_2, x_3. From the relation obtained in this way, considering (3.1.25), it follows by permutation of indices $l \Leftrightarrow m$ and division by λ.

$$\chi_m e_k \delta_{kl} + \chi_k e_m \delta_{ml} + 2\chi_l e_m \delta_{mk} = 2\phi_l e_k \delta_{km} \tag{3.1.27}$$

where

$$\phi_l = \frac{\Phi_l}{\lambda} \tag{3.1.28}$$

Because the relation (3.1.27) must be identically satisfied for all the values of k, l and m, there results, especially for $k = m$, $m \neq l$,

$$\chi_l = \phi_l \qquad (l = 0, 1, 2, 3) \tag{3.1.29}$$

and also for $k = l$

$$\chi_m = 0 \tag{3.1.30}$$

Consequently we have

$$\chi_l = \phi_l = 0 \tag{3.1.31}$$

Then from (3.1.26), (3.1.28) and (3.1.31) one gets $\partial\lambda/\partial x_m = 0$, i.e.

$$\lambda = \lambda(v) \tag{3.1.32}$$

and from (3.1.25) and (3.1.30), for the functions f_i, the relation

$$\frac{\partial^2 f_i}{\partial x_k \, \partial x_l} = 0 \tag{3.1.33}$$

This is the second condition for the functions f_i. On the basis of these considerations it follows finally that the functions f_i satisfying the conditions (3.1.18) and (3.1.33) must necessarily be linear

$$x_i' = f_i(v, x_0, x_1, x_2, x_3) = \sqrt{[\lambda(v)]}\,(e_k a_{ik} x_k + C_i) \tag{3.1.34}$$

C_i being additive constants. All these constants C_i vanish if we suppose that the coordinate origins of both the Galilean systems of reference S and S' coincide at the moment $t = 0$. In this case we have

$$x_i' = f_i(v, x_0, x_1, x_2, x_3) = \sqrt{[\lambda(v)]}\, e_k a_{ik} x_k \tag{3.1.35}$$

For the inverse transformations one obtains

$$x_i = \frac{1}{\sqrt{\lambda}} e_k a_{ki} x_k' \tag{3.1.36}$$

Consequently the linearity of transformation is a theorem within our theory which is based on the axioms A_1, A_2, A_3 and A_4.

3.2. *The Law of the Constancy of the Velocity of Light*

It may be pointed out that the 'principle' of the constancy of the velocity of light, introduced by Einstein as a hypothesis, is composed of two assertions. The first of these tells us that in any Galilean system of reference the velocity of light *in vacuo* is the same in all directions and the second means that this velocity has always the same amount in every such system independently of the velocity of the source of light. From (3.1.18) and (3.1.33) as well from (3.1.11) and (3.1.13) and (3.1.34) for the coefficients of the linear transformation there result the relations

$$e_i a_{ik} a_{il} = e_k \delta_{kl} \tag{3.2.1}$$

$$e_i a_{ki} a_{li} = e_k \delta_{kl} \tag{3.2.2}$$

In the system S' we take a point which is at rest. Relative to an observer in the system S this point is moving with a constant velocity, the components of which we shall denote by v_1, v_2 and v_3. Then

$$v_i = \frac{dx_i}{dt} = c\frac{dx_i}{dx_0} \qquad \text{for } \dot{x}_1' = \dot{x}_2' = \dot{x}_3' = 0 \tag{3.2.3}$$

Denoting by v_1', v_2' and v_3' the components of the velocity of a point being at rest in the system S, then, in the system S', we have

$$v_i' = \frac{dx_i'}{dt'} = c'\frac{dx_i'}{dx_0'} \qquad \text{for } \dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0 \tag{3.2.4}$$

From (3.1.34) and (3.2.3) and (3.1.33) and (3.2.4) there follows respectively

$$\frac{a_{0i}}{a_{00}} = \frac{v_i}{c} \tag{3.2.5}$$

$$\frac{a_{i0}}{a_{00}} = \frac{v_i'}{c'} \tag{3.2.6}$$

c' being the velocity of light in the system S'. We do not know *a priori* if it equals the constant c of the velocity of light in the system S.

From (3.2.1) and (3.2.2) for $k = l = 0$ there results

$$a_{00}^2 - (a_{10}^2 + a_{20}^2 + a_{30}^2) = 1 \tag{3.2.7}$$

$$a_{00}^2 - (a_{01}^2 + a_{02}^2 + a_{03}^2) = 1 \tag{3.2.8}$$

and therefrom with the aid of the relations (3.2.5) and (3.2.6)

$$\frac{1}{c'^2}(v_1'^2 + v_2'^2 + v_3'^2) = \frac{1}{c^2}(v_1{}^2 + v_2{}^2 + v_3{}^2) \tag{3.2.9}$$

Assume for a moment that

$$v_1 = v \qquad v_2 = v_3 = 0 \tag{3.2.10}$$

and that the X'-axis of the Galilean system of reference S' coincides with the X-axis of the system S. The Y'- and Z'-axis shall be parallel to the Y- and Z-axis respectively. For the points of the system S', for which $\dot{x}' = \dot{y}' = \dot{z}' = 0$, it will be (in the system S) $\dot{x} = v$, $\dot{y} = \dot{z} = 0$, while for the points of the system S, for which $\dot{x} = \dot{y} = \dot{z} = 0$, according to the axiom A_4 the relations

$$\dot{x}' = v_1' = v' = -v \qquad \dot{y} = v_2' = 0 \qquad \dot{z}' = v_3' = 0 \tag{3.2.11}$$

are valid.

From (3.2.9) and (3.2.11) there follows then, necessarily, the fundamental relation

$$c = c' \tag{3.2.12}$$

That means: The velocity of light *in vacuo* is independent of the relative velocity of the Galilean systems of reference S and S' and represents consequently a universal constant. This is the 'principle' of constancy of the velocity of light which Einstein has originally taken as a fundamental hypothesis of his theory. In our theory, based on the set of axioms A_1, A_2, A_3 and A_4, the proposition (3.2.12) is a theorem.

3.3. *The Determination of the Coefficients of the Transformation* (3.2.1)

From (3.2.1), (3.2.2), (3.2.5), (3.2.6) and (3.2.12) one can determine the coefficients of the transformation (3.1.35) and (3.1.36) in the known way (Fock, 1960b). These coefficients are then

$$a_{00} = \frac{1}{\sqrt{[1-(v^2/c^2)]}}$$

$$a_{0i} = \frac{v_i/c}{\sqrt{[1-(v^2/c^2)]}} \qquad a_{i0} = -\frac{\sum_{l=0}^{3} \alpha_{il}(v_l/c)}{\sqrt{[1-(v^2/c^2)]}} \tag{3.3.1}$$

$$a_{ik} = -\alpha_{ik} - \left(\frac{1}{\sqrt{[1-(v^2/c^2)]}} - 1\right)\frac{v_k}{v^2}\sum_{l=1}^{3} \alpha_{il} v_l$$

where $i, k = 1, 2, 3$.

The quantities α_{ik} in (3.3.1) are the cosines of the angles between the old and the new coordinate axes where the first index i relates to the new and the second index k to the old ones. Taking that in the two Galilean systems S and S' the X- and X'-axes coincide permanently and that the axes Y and Z are permanently parallel to the axes Y' and Z' respectively, for the transformation connecting the coordinates of the both systems S and S' we obtain

$$x' = \sqrt{(\lambda)}\, k(x - vt) \quad y' = \sqrt{(\lambda)}\, y \quad z' = \sqrt{(\lambda)}\, z \quad t' = \sqrt{(\lambda)}\, k\left(t - \frac{vx}{c^2}\right)$$

where

$$\lambda = \lambda(v) \qquad k = \frac{1}{\sqrt{[1-(v^2/c^2)]}} \tag{3.3.2}$$

A very relevant consequence of our theory is the result that the axioms A_1, A_2, A_3 and A_4 lead to a more general transformation (3.3.2) (since $\lambda = \lambda(v)$ is an arbitrary function of v), which leaves the Maxwell equations for the electromagnetic field invariant and contains the Lorentz and the Palacios transformation (Palacios, 1960) as special cases for $\lambda(v) = 1$ and $\lambda(v) = [1 - (v^2/c^2)]^{-1}$ respectively. Thus the Lorentz transformation is not the unique transformation leaving the Maxwell equations for the electromagnetic field invariant with respect to the Galilean systems of reference as Einstein considered, but there exists a large class of transformations having this property.

The Lorentz transformation follows only in the case if to the axioms A_1, A_2, A_3 and A_4 the following axiom of Einstein is added:

A_5. Let S and S' be two Galilean systems of reference moving relatively to each other with a definite velocity $+v$ or $-v$ along the X- and X'-axis, which coincide, such that the Y- and Y'-axis and the Z- and Z'-axis are parallel. The distance of any two points at rest in S, situated in a plane orthogonal to the X-axis, is relative to the Galilean system of reference S' independent of the direction of the velocity. The same is true for any two points at rest in S', situated in a plane orthogonal to the X'-axis.

Only in this case it follows that $\lambda = \lambda(v) = \lambda(-v)$ and $\lambda(v)\,\lambda(-v) = 1$ and consequently $\lambda = 1$ (Pauli, 1921).

Our general transformation (3.3.2) leads to the known formulae for the transformation of the components of velocities in the system S and S'

$$\dot{x}' = \frac{\dot{x} - v}{1 - (v\dot{x}/c^2)} \qquad \dot{y}' = \frac{\dot{y}\sqrt{[1 - (v^2/c^2)]}}{1 - (v\dot{x}/c^2)} \qquad \dot{z}' = \frac{\dot{z}\sqrt{[1 - (v^2/c^2)]}}{1 - (v\dot{x}/c^2)} \qquad (3.3.3)$$

(3.3.3) being the common formulae for the special theory of relativity and for the Palacios theory (Stiegler, 1971).

3.4. *A View that the Michelson–Morley Experiment does not Act as a Proof of the Validity of the Theory of Special Relativity*

It is interesting to point out that the fundamental relation (3.2.12) expressing the so called 'principle' of the constancy of the velocity of light as well as the transformation (3.3.2)—containing the Lorentz and the Palacios transformation as special cases—are logical consequences (theorems) of the theory which is based on the axioms A_1, A_2, A_3 and A_4. We know that the negative result of the Michelson–Morley experiment can be explained by means of (3.2.12), i.e. on the basis of the 'principle' of the constancy of the velocity of light. But, since the relation $c = c'$ is a consequence (theorem) of the theory based on the axioms A_1, A_2, A_3 and A_4, it is a proposition of a theory which is more general than the theory of special relativity and Palacios theory being a common theorem of both these theories. Thus the proposition (3.2.12), i.e. the fundamental relation $c = c'$, is contained not only in the Lorentz but also in the Palacios transformation, which are special cases of the general transformation (3.3.2), differing basically the one from the other. Consequently the Michelson–Morley experiment does not represent an experimental proof in favour of the theory of special relativity.

3.5. *Some General Remarks Concerning the Theoretical Explanations of the Fizeau Experiment, Doppler Effect and Aberration of Light*

It is not difficult to see that with the aid of our general transformation (3.3.2) one gets the well-known expression for the velocity of light in moving water

$$u_x = \dot{x} = \frac{c}{n'} + v\left(1 - \frac{1}{n'^2}\right)$$

in agreement with the result obtained by Fizeau. Consequently the Fizeau experiment does not represent an experimental proof in favour of the theory of special relativity. On the other hand, (3.3.2) gives for the general Doppler effect the expressions

$$\nu' = \frac{1}{\sqrt{\lambda}} \cdot \nu \frac{1 - (v/c)\cos\alpha}{\sqrt{[1 - (v^2/c^2)]}}$$

$$\cos\alpha' = \frac{\cos\alpha - (v/c)}{1 - (v/c)\cos\alpha} \qquad \cos\beta' = \frac{\cos\beta\sqrt{[1 - (v^2/c^2)]}}{1 - (v/c)\cos\alpha}$$

$$\cos\gamma' = \frac{\cos\gamma\sqrt{[1 - (v^2/c^2)]}}{1 - (v/c)\cos\alpha}$$

Denoting by δ the angle between the old and the new direction of the wave-normal, then respecting the above expressions, for the variation of the direction of the wave-normal we get the known expression

$$\cos\delta = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$$
$$= \frac{\cos\alpha\,[\cos\alpha - (v/c)] + (\cos^2\beta + \cos^2\gamma)\sqrt{[1 - (v^2/c^2)]}}{1 - (v/c)\cos\alpha}$$

The Palacios theory, corresponding to the special case $\lambda(v) = [1 - (v^2/c^2)]^{-1}$, gives for the Doppler effect the original expression $\nu' = \nu[1 - (v/c)\cos\alpha]$ and for the aberration of light $[\alpha = (\pi/2)]$ the expression $\nu' = \nu$, while the Lorentz transformation, corresponding to the special case $\lambda(v) = 1$, conduces to the Einstein expression

$$\nu' = \nu\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx \nu\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

representing an effect of the second order.

3.6. *The Principle of Relativity and the Discernibility and the Indiscernibility of 'Right' and 'Left'*

Take now three Galilean systems of reference S, S' and S'' such that S' relative to S and S'' relative to S' moves with the velocity v. Let, x, y, z, t

be coordinates and time; X, Y, Z the components of the vector of the electric and L, M, N the components of the vector of the magnetic field relative to S. Let further x', y', z', t' and x'', y'', z'', t'' be coordinates and times, then X', Y', Z' and X'', Y'', Z'' and L', M', N' and L'', M'', N'' the components of the vector of the electric and of the magnetic field in S' and S'' respectively.

The coordinates and times of S' and S on one and S'' and S' on the other hand are connected by the general transformation

$$\begin{aligned} x' &= \sqrt{[\lambda(v)]}\,k(x-vt) \\ y' &= \sqrt{[\lambda(v)]}\,y \\ z' &= \sqrt{[\lambda(v)]}\,z \\ t' &= \sqrt{[\lambda(v)]}\,k\left(t-\frac{vx}{c^2}\right) \end{aligned} \tag{3.6.1}$$

and

$$\begin{aligned} x'' &= \sqrt{[\lambda(v)]}\,k(x'-vt') \\ y'' &= \sqrt{[\lambda(v)]}\,y' \\ z'' &= \sqrt{[\lambda(v)]}\,z' \\ t'' &= \sqrt{[\lambda(v)]}\,k\left(t'-\frac{vx'}{c^2}\right) \end{aligned} \tag{3.6.2}$$

respectively, with

$$k = \frac{1}{\sqrt{[1-(v^2/c^2)]}}$$

For the Maxwell's equations in S we have

$$\begin{aligned} &\frac{1}{c}\frac{\partial X}{\partial t} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z} \qquad && -\frac{1}{c}\frac{\partial L}{\partial t} = \frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z} \\ &\frac{1}{c}\frac{\partial Y}{\partial t} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x} && -\frac{1}{c}\frac{\partial M}{\partial t} = \frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x} \\ &\frac{1}{c}\frac{\partial Z}{\partial t} = \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} && -\frac{1}{c}\frac{\partial N}{\partial t} = \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} \\ &\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 0 && \frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = 0 \end{aligned} \tag{3.6.3}$$

In accordance with the principle of relativity (A_1) in the system S' it will be

$$\begin{aligned} &\frac{1}{c}\frac{\partial X'}{\partial t'} = \frac{\partial N'}{\partial y'} - \frac{\partial M'}{\partial z'} \qquad && -\frac{1}{c}\frac{\partial L'}{\partial t'} = \frac{\partial Z'}{\partial y'} - \frac{\partial Y'}{\partial z'} \\ &\frac{1}{c}\frac{\partial Y'}{\partial t'} = \frac{\partial L'}{\partial z'} - \frac{\partial N'}{\partial x'} && -\frac{1}{c}\frac{\partial M'}{\partial t'} = \frac{\partial X'}{\partial z'} - \frac{\partial Z'}{\partial x'} \\ &\frac{1}{c}\frac{\partial Z'}{\partial t'} = \frac{\partial M'}{\partial x'} - \frac{\partial L'}{\partial y'} && -\frac{1}{c}\frac{\partial N'}{\partial t'} = \frac{\partial Y'}{\partial x'} - \frac{\partial X'}{\partial y'} \\ &\frac{\partial X'}{\partial x'} + \frac{\partial Y'}{\partial y'} + \frac{\partial Z'}{\partial z'} = 0 && \frac{\partial L'}{\partial x'} + \frac{\partial M'}{\partial y'} + \frac{\partial N'}{\partial z'} = 0 \end{aligned} \tag{3.6.4}$$

and in the system S''

$$\begin{aligned}
&\frac{1}{c}\frac{\partial X''}{\partial t''}=\frac{\partial N''}{\partial y''}-\frac{\partial M''}{\partial z''} && -\frac{1}{c}\frac{\partial L''}{\partial t''}=\frac{\partial Z''}{\partial y''}-\frac{\partial Y''}{\partial z''}\\
&\frac{1}{c}\frac{\partial Y''}{\partial t''}=\frac{\partial L''}{\partial z''}-\frac{\partial N''}{\partial x''} && -\frac{1}{c}\frac{\partial M''}{\partial t''}=\frac{\partial X''}{\partial z''}-\frac{\partial Z''}{\partial x''}\\
&\frac{1}{c}\frac{\partial Z''}{\partial t''}=\frac{\partial M''}{\partial x''}-\frac{\partial L''}{\partial y''} && -\frac{1}{c}\frac{\partial N''}{\partial t''}=\frac{\partial Y''}{\partial x''}-\frac{\partial X''}{\partial y''}\\
&\frac{\partial X''}{\partial x''}+\frac{\partial Y''}{\partial y''}+\frac{\partial Z''}{\partial z''}=0 && \frac{\partial L''}{\partial x''}+\frac{\partial M''}{\partial y''}+\frac{\partial N''}{\partial z''}=0
\end{aligned} \tag{3.6.5}$$

From (3.6.1) and (3.6.3) we get:

$$\begin{aligned}
&\frac{1}{c}\frac{\partial X}{\partial t'}=\frac{\partial k[N-(v/c)\,Y]}{\partial y'}-\frac{\partial k[M+(v/c)Z]}{\partial z'}\\
&\frac{1}{c}\frac{\partial k[Y-(v/c)N]}{\partial t'}=\frac{\partial L}{\partial z'}-\frac{\partial k[N-(v/c)\,Y]}{\partial x'}\\
&\frac{1}{c}\frac{\partial k[Z+(v/c)M]}{\partial t'}=\frac{\partial k[M+(v/c)Z]}{\partial x'}-\frac{\partial L}{\partial y'}\\
&-\frac{1}{c}\frac{\partial L}{\partial t'}=-\frac{\partial k[Y-(v/c)N]}{\partial z'}+\frac{\partial k[Z+(v/c)M]}{\partial y'}\\
&-\frac{1}{c}\frac{\partial k[M+(v/c)Z]}{\partial t'}=-\frac{\partial k[Z+(v/c)M]}{\partial x'}+\frac{\partial X}{\partial z'}\\
&-\frac{1}{c}\frac{\partial k[N-(v/c)\,Y]}{\partial t'}=-\frac{\partial X}{\partial y'}+\frac{\partial k[Y-(v/c)N]}{\partial x'}
\end{aligned} \tag{3.6.6}$$

where

$$k=\frac{1}{\sqrt{[1-(v^2/c^2)]}}$$

In an analogous way to the above, from (3.6.2) and (3.6.4) it follows that

$$\begin{aligned}
&\frac{1}{c}\frac{\partial X'}{\partial t''}=\frac{\partial k[N'-(v/c)\,Y']}{\partial y''}-\frac{\partial k[M'+(v/c)Z']}{\partial z''}\\
&\frac{1}{c}\frac{\partial k[Y'-(v/c)N']}{\partial t''}=\frac{\partial L'}{\partial z''}-\frac{\partial k[N'-(v/c)\,Y']}{\partial x''}\\
&\frac{1}{c}\frac{\partial k[Z'+(v/c)M']}{\partial t''}=\frac{\partial k[M'+(v/c)Z']}{\partial x''}-\frac{\partial L'}{\partial y''}\\
&-\frac{1}{c}\frac{\partial L'}{\partial t''}=-\frac{\partial k[Y'-(v/c)N']}{\partial z''}+\frac{\partial k[Z'+(v/c)M']}{\partial y''}\\
&-\frac{1}{c}\frac{\partial k[M'+(v/c)Z']}{\partial t''}=-\frac{\partial k[Z'+(v/c)M']}{\partial x''}+\frac{\partial X'}{\partial z''}\\
&-\frac{1}{c}\frac{\partial k[N'-(v/c)\,Y']}{\partial t''}=-\frac{\partial X'}{\partial y''}+\frac{\partial k[Y'-(v/c)N']}{\partial x''}
\end{aligned} \tag{3.6.7}$$

By comparison of (3.6.4) and (3.6.6) we get

$$\begin{aligned} X' &= \sqrt{[\lambda(v)]}\, X & L' &= \sqrt{[\lambda(v)]}\, L \\ Y' &= \sqrt{[\lambda(v)]}\, k\left(Y - \frac{v}{c} N\right) & M' &= \sqrt{[\lambda(v)]}\, k\left(M + \frac{v}{c} Z\right) \\ Z' &= \sqrt{[\lambda(v)]}\, k\left(Z + \frac{v}{c} M\right) & N' &= \sqrt{[\lambda(v)]}\, k\left(N - \frac{v}{c} Y\right) \end{aligned} \tag{3.6.8}$$

and also by comparison of (3.6.5) and (3.6.7)

$$\begin{aligned} X'' &= \sqrt{[\lambda(v)]}\, X' & L'' &= \sqrt{[\lambda(v)]}\, L' \\ Y'' &= \sqrt{[\lambda(v)]}\, k\left(Y' - \frac{v}{c} N'\right) & M'' &= \sqrt{[\lambda(v)]}\, k\left(M' + \frac{v}{c} Z'\right) \\ Z'' &= \sqrt{[\lambda(v)]}\, k\left(Z' + \frac{v}{c} M'\right) & N'' &= \sqrt{[\lambda(v)]}\, k\left(N' - \frac{v}{c} Y'\right) \end{aligned} \tag{3.6.9}$$

Taking now especially that S'' is moving relative to S' with the velocity $-v$, then S'' will be at rest relative to S. In this case it follows from (3.6.9)

$$\begin{aligned} X'' &= \sqrt{[\lambda(-v)]}\, X' \\ Y'' &= \sqrt{[\lambda(-v)]}\, k\left(Y' + \frac{v}{c} N'\right) \\ Z'' &= \sqrt{[\lambda(-v)]}\, k\left(Z' - \frac{v}{c} M'\right) \\ L'' &= \sqrt{[\lambda(-v)]}\, L' \\ M'' &= \sqrt{[\lambda(-v)]}\, k\left(M' - \frac{v}{c} Z'\right) \\ N'' &= \sqrt{[\lambda(-v)]}\, k\left(N' + \frac{v}{c} Y'\right) \end{aligned} \tag{3.6.10}$$

Taking into account that S'' is at rest relative to S, it follows that

$$X'' = X \quad Y'' = Y \quad Z'' = Z \quad L'' = L \quad M'' = M \quad N'' = N \tag{3.6.11}$$

Then from (3.6.10) and (3.6.8) it results

$$\lambda(v) . \lambda(-v) = 1 \tag{3.6.12}$$

From the relation (3.6.12) it follows

$$\lambda(-v) = \frac{1}{\lambda(v)} \tag{3.6.13}$$

Taking $v \Rightarrow -v$ in (3.6.8) we get

$$\begin{aligned} X' &= \sqrt{[\lambda(-v)]}\, X & L' &= \sqrt{[\lambda(-v)]}\, L \\ Y' &= \sqrt{[\lambda(-v)]}\, k\left(Y + \frac{v}{c} N\right) & M' &= \sqrt{[\lambda(-v)]}\, k\left(M - \frac{v}{c} Z\right) \\ Z' &= \sqrt{[\lambda(-v)]}\, k\left(Z - \frac{v}{c} M\right) & N' &= \sqrt{[\lambda(-v)]}\, k\left(N + \frac{v}{c} Y\right) \end{aligned} \tag{3.6.14}$$

and respecting (3.6.13)

$$X' = \frac{1}{\sqrt{[\lambda(v)]}} X \qquad L' = \frac{1}{\sqrt{[\lambda(v)]}} L$$
$$Y' = \frac{1}{\sqrt{[\lambda(v)]}} k\left(Y + \frac{v}{c} N\right) \quad M' = \frac{1}{\sqrt{[\lambda(v)]}} k\left(M - \frac{v}{c} Z\right) \qquad (3.6.15)$$
$$Z' = \frac{1}{\sqrt{[\lambda(v)]}} k\left(Z - \frac{v}{c} M\right) \quad N' = \frac{1}{\sqrt{[\lambda(v)]}} k\left(N + \frac{v}{c} Y\right)$$

By a comparison of (3.6.8) with (3.6.15), we see that changing the direction of the relative velocity $v \Rightarrow -v$, i.e. changing the 'left' and the 'right', the components of the vector of the electric and the magnetic field will be changed, i.e. the 'right' is not equivalent to the 'left' and vice versa. That means that the validity of the relation (3.6.12) is equivalent with the *discernibility of the 'left' from the 'right'*. We see further, that changing $v \Rightarrow -v$, this difference between the 'right' and the 'left' disappears, if we suppose the validity of the axiom A_5. In this case $y' = y$, $z' = z$, and from (3.6.1) it follows that $\lambda(v) = 1$ and respecting (3.6.13) also $\lambda(-v) = 1$, i.e.,

$$\lambda(v) = \lambda(-v) \qquad (3.6.16)$$

The axiom A_5 or the relation (3.6.16) is thus equivalent to the *indiscernibility* of 'right' and 'left'.

Further, we see also that the validity of the principle of relativity, i.e. the axiom A_1, together with A_2, is *compatible* with the existence of the relation (3.6.12) and (3.6.16), i.e. with the *discernibility as well as with the indiscernibility of 'right' and 'left' in the physical space.*

But the indiscernibility of 'right' and 'left', i.e. the validity of the axiom A_5 or, equivalently, the relations (3.6.12) and (3.6.16), is *equivalent* to the existence of the Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{[1 - (v^2/c^2)]}} \quad y' = y \quad z' = z \quad t' = \frac{t - (vx/c^2)}{\sqrt{[1 - (v^2/c^2)]}} \qquad (3.6.17)$$

On the other hand, the assumption of the validity of the discernibility of 'right' and 'left' (i.e. the assumption that the axiom A_5 is not valid!) *is equivalent* to the existence of the general transformation (3.3.2), which contains the Palacios transformation as a special case for

$$\lambda(v) = [1 - (v^2/c^2)]^{-1}.$$

3.7. *To Decide the Question of the Discernibility or Indiscernibility of the 'Right' from the 'Left' at the Macrocosmic Level*

We come now to the very important question of whether in macrocosmical reality the indiscernibility of 'right' from 'left' is valid or not. This question can be solved only by experiments. We know, for example, that the theory of aberration of light shows that the expression

$$\nu' = \frac{\nu}{\sqrt{[1 - (v^2/c^2)]}} \qquad (3.7.1)$$

is valid exactly if and only if the Lorentz transformation (3.6.17) holds exactly. In reality, the experiments do not agree exactly with (3.7.1) but only with the approximate expression $\nu' = \nu[1 + \frac{1}{2}(v^2/c^2)]$. In fact, this approximate relation does not correspond to the Lorentz transformation with $\lambda(v) = 1$, but to a transformation of the form (3.3.2) $\lambda(v)$ being generally a function of v/c, which is, as we already know, equivalent to the validity of *discernibility* of 'right' from 'left' at the macrocosmic level.

The postulate (principle) of the *indiscernibility* of 'right' and 'left', i.e. the space-reflection invariance, was questioned for the first time in 1956 by Yang and Lee in connection with the theory of weak interactions. The *violation* of this 'principle' in connection with weak interactions has been proved experimentally by Wu, Schopper and others.

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